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Appearance of an artificial length scale in JETSET Bose-Einstein simulation

Raluca Mureşan[a], Oxana Smirnova[b], Bengt Lörstad

Department of Elementary Particle Physics, Lund University, Box 118, SE-22100 Lund, Sweden

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Abstract. This work studies the algorithm which implements the Bose-Einstein correlation effect in the JETSET 7.4 event generator. This algorithm attempts to reproduce an expected correlation function with a given correlation radius and amplitude. The two-particle correlation function is studied in the generated Z^0 hadronic decays for different values of the built-in radius parameter. Samples consisting of only charged particles are used, as well as subsamples of pions, pions coming from the string decay and pions from the resonance decays. The Bose-Einstein correlation function, extracted from the generated events, is parameterized analogously to the built-in JETSET correlation function and its parameters are compared with the input ones. We found that the measured correlation function reproduces the built-in one, if the input radius parameter is larger than $1\,fm$. For lower input radii an artificial new length scale appears due to the way the Bose-Einstein correlation is implemented.

1 Introduction

The study of Bose-Einstein correlation function for identical particles is of particular interest, since the process of hadron production, or fragmentation, in high energy physics is poorly comprehended. At this moment, no appropriate theory can describe it, only phenomenological models being available for the hadronization process and the Bose-Einstein phenomenon in particular. To account for and to describe it properly in event generators, the Bose-Einstein phenomenon must be well understood, therefore, more profound studies are required. Recently it was shown that the measurement of W mass at LEP2 is likely to be affected by Bose-Einstein correlation between pions from different W's [1–3]. Consequently, a significant interest was shown for the study of Bose-Einstein correlations in e^+e^- annihilation in the last years [4–7]. Comparison of the LEP data with the analysis of the $e^+e^- \to Z^0$ events, generated with the JETSET [8] particle generator with the built-in algorithm for the Bose-Einstein correlation simulation, showed that the model reproduces experimental data very well [6, 9–11].

Since JETSET is the simulation program the most commonly used by e^+e^- annihilation experimental groups, the analysis of the built-in method of implementing the Bose-Einstein correlations[1], its performance, its drawbacks, and the possible improvements were studied in several works [12, 13]. Our aim is to clarify the dependence of the correlation function on assumed input correlation radius and the particle sample composition.

The paper is organised as follows: in Sect. 2 the theory of the Bose-Einstein correlations is presented. Section 3 is dedicated to the description of the JETSET simulation program and the way in which the Bose-Einstein algorithm is implemented. Implementation issues and practical hints for our analysis are presented in Sect. 4. In Sect. 5 the behaviour of the correlation function for particles with different origin is studied and in Sect. 6 its dependence on the input correlation radius is investigated. Finally, in Sect. 7 the reader finds the conclusions.

2 Bose-Einstein correlation

During the 50-ies, in particle physics experiments, it was discovered that the produced bosons show a tendency to have close energy-momentum characteristics [14]. This phenomenon of increasing probability for emission of identical bosons from close regions of space and time is called Bose-Einstein correlation (we will not discuss correlations between fermions in this paper).

The Bose-Einstein effect originates in the quantum mechanical interference of the boson wave functions. It is a consequence of their symmetry under particle exchange, that influences the wave functions to yield an effective

[a] On leave from the National Institute for Research and Development in Nuclear Physics and Engineering "Horia Hulubei" (IFIN-HH) RO-76 900 P.O. Box MG 6, Bucharest, Romania

[b] On leave from JINR, Dubna, Moscow district, 141980 Russia

[1] JETSET is so far the only e^+e^- generator that contains a built-in Bose-Einstein correlation algorithm

clustering of the particles in the phase space, which explains their preference for occupying the same quantum states. From the characteristics of the resulting interference pattern it is possible, at least in principle, to determine the space-time dimensions of the source.

The string model describes e^+e^- annihilation data with high accuracy and provides an appropriate framework in which to consider the Bose-Einstein enhancement. In spite of a few attempts to introduce the Bose-Einstein symmetrization into the string model [15,16], the only working method to introduce the interference effects in Monte Carlo generators for e^+e^- annihilation, so far, is the shifting of the final state momenta to reproduce the assumed distribution of some variables [8]. This technique is described in details in Sect. 3. It has to be stressed that such a method has nothing to do with the quantum mechanics and only simulates the expected effect.

2.1 The Bose-Einstein correlation function

Considering the production of two identical bosons with four-momenta p_1 and p_2, denoting $P(p_1, p_2)$ the probability density of two particles to be produced, and $P(p_1)$ and $P(p_2)$ as the probability densities for a single particle to be produced with momentum p_1 or p_2, the correlation function C_2 of two identical bosons is defined as [17]:

$$C_2(p_1, p_2) = \frac{P(p_1, p_2)}{P(p_1)P(p_2)} \ . \tag{1}$$

From the experimental point of view, $P(p_1, p_2)$ is a double differential cross section. In practice, it is difficult to construct the product $P(p_1)P(p_2)$ due to the phase space limitations, therefore it is often replaced by $P_0(p_1, p_2)$, which is equal to $P(p_1)P(p_2)$ in the absence of correlation. One of the major problems in these kinds of studies is how to build $P_0(p_1, p_2)$, usually called the "reference sample".

The correlation function (1) is often parameterized as [17]:

$$C_2(Q) = N(1 + \lambda e^{-Q^2R^2}) \ , \tag{2}$$

where $Q = p_1 - p_2$ is the invariant four-momenta difference, and N, λ, R are free parameters. N is a normalisation constant. The interpretation of the λ parameter is that it is related to the fraction of identical bosons which do interfere, effectively representing the correlation strength. Parameter R is usually interpreted as the geometrical radius of a presumably spherical boson emitting source, or simply as the correlation radius parameter.

Near $Q = 0$, the effect of Coulomb repulsion between two identical charged bosons will lead to the suppression of the probability of finding two like-charged particles with small relative momentum. The effect is dominant in case of pions for $Q < 10$ MeV. For $Q > 50$ MeV the Coulomb repulsion is negligible. Since it has only a small effect on the studied distributions in e^+e^- annihilation, it is not discussed in the present article.

3 JETSET and Bose-Einstein correlation

JETSET is a simulation program able to generate hard processes, in particular, the electron-positron annihilation producing a boson, which decays into a quark-antiquark pair: $e^+e^- \to \gamma^*/Z^0 \to q\bar{q}$ (here '*' is used to denote that the photon is off-mass-shell). In what follows, events of this kind will be referred to as "$q\bar{q}$ events". The quark q in the reaction may have any flavour, picked at random, according to the relative couplings evaluated at the hadronic center of mass energy.

JETSET is intimately connected with the string fragmentation model in the form of the so-called Lund model. The JETSET program has a probabilistic and iterative nature, with the fragmentation process being described in terms of one or few simple underlying branches, as, for example, *string* $\to$ *hadron* + *remainder string* and so on. At each branching probabilistic rules are given for production of new flavours and for sharing of the energy and momentum between the products.

To understand the fragmentation model, we can use as an example the simplest possible system: a 2-jet colour singlet $q\bar{q}$ event, as produced in e^+e^- annihilation, where we have a linear confinement picture. The energy stored in the colour dipole field between a charge and an anticharge increases linearly with the separation between charges, if the Coulomb term is neglected. This assumption of the linear confinement provides the starting point for the string model. As q and $\bar{q}$ partons move apart from their common production vertex, a colour flux tube or maybe a colour vortex line is considered to be stretched between the $q\bar{q}$. The potential energy stored inside the string increases and the string may break, producing a new $q'\bar{q}'$ pair, so that the system splits into two colour singlet systems $q'\bar{q}$ and $q\bar{q}'$. If the invariant mass of these pairs is large enough, further breaks may occur. The generator does not take into account either the propagation of the resonances, or the space-time picture of the particle creation. The string break-up process is assumed to proceed until only the on-mass-shell hadrons remain, each hadron corresponding to a small piece of string with a quark at one end and an antiquark at the other.

To simulate the Bose-Einstein effect, an algorithm, which does not represent a true model, is used, for which very specific assumptions and choices are made. In this scheme the fragmentation is allowed to proceed independently of Bose-Einstein effect. The four-momentum difference, Q_{ij}, associated to a pair of identical particles i, j is defined as:

$$Q_{ij} = \sqrt{(p_i + p_j)^2 - 4m^2} \ , \tag{3}$$

where m is the common particle mass and p_i, p_j are particle momenta.

A shifted smaller Q'_{ij} is then sought, such that the ratio of "shifted" to the "unshifted" Q distribution is given by the requested parameterization $C_2(Q)$. The shape can be chosen to be either exponential or Gaussian :

$$C_2(Q) = 1 + \lambda e^{-(QR)^r}, \quad r = 1 \ or \ 2 \ , \tag{4}$$

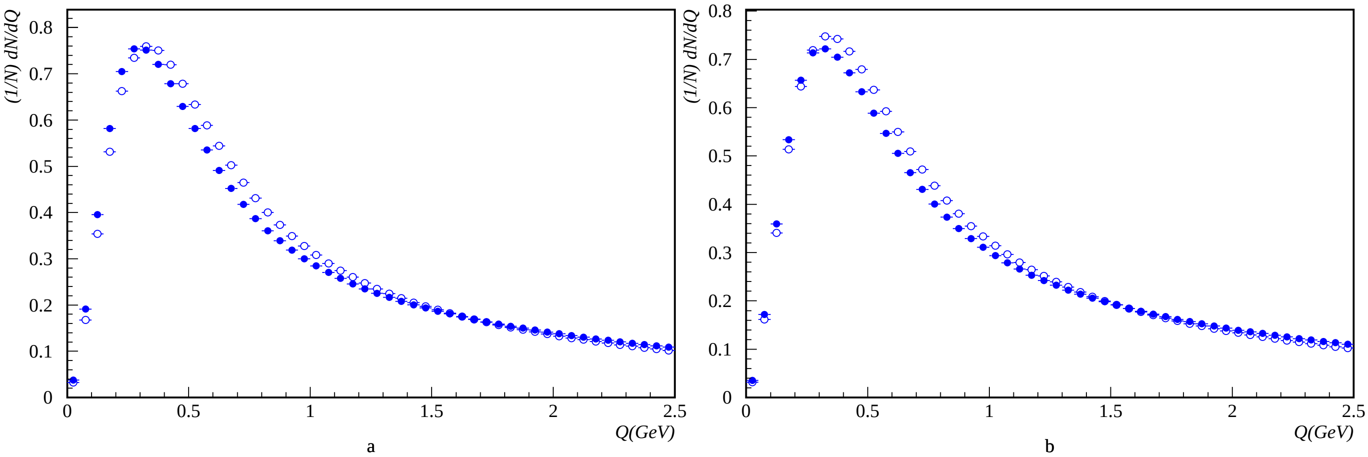

Fig. 1. The stored Q histograms (normalised to unity), for all charged particles, 2 millions JETSET events: **a** Bose-Einstein correlation effect included, but the mixing procedure not performed: closed circles, Bose-Einstein correlation effect included, and the mixing procedure performed: open circles, **b** Bose-Einstein effect not included, and the mixing procedure performed: open circles, Bose-Einstein effect not included, and the mixing procedure not performed: closed circles

where λ, R and r are input parameters of the algorithm (we have used in this work only the Gaussian form, $r = 2$).

If the inclusive distribution of the Q_{ij} is assumed to be given just by the simple, spherical phase space, Q'_{ij} can be found as a solution of the equation:

$$\int_0^{Q_{ij}} \frac{Q^2 dQ}{\sqrt{Q^2+4m^2}} = \int_0^{Q'_{ij}} C_2(Q) \frac{Q^2 dQ}{\sqrt{Q^2+4m^2}} \ . \tag{5}$$

That gives as a new distribution Q'_{ij} – the product of the old distribution Q_{ij} and $C_2(Q)$:

$$Q'_{ij} = Q_{ij} C_2(Q) \ . \tag{6}$$

This procedure is performed for all the pairs of identical bosons (for resonances with a width above a certain value - in our case $0.020 GeV$ - the decays are assumed to take place before the stage where Bose-Einstein effects are introduced). The new values for Q are calculated and a global shift of momenta for all the particles is performed adding all the possible pair shifts. Finally, all the momenta are rescaled by a common factor to restore the original value of the total centre of mass energy. The obtained invariant Q distribution is in this way shifted, see Fig. 1. This algorithm is implemented in the LUBOEI subroutine, a standard component of the JETSET generator. The assumption of the simple spherical phase space is valid only for the part of the true Q distribution to the left of the peak, see Fig. 1. Instead of a monotonic increase, which is given by the spherical phase space, the experimental Q distribution has a maximum around 0.5 GeV, then falls power-like for large Q. Fiałkowski and Wit [12] and Haywood [13] proposed modified procedures using approximations of the experimental Q distributions to estimate the Q shift. However, using these methods, the final shifted Q distribution did not correspond to the input values of R parameters as in (4), for $R \leq 1\, fm$.

4 Computational program: main assumptions

We used JETSET in order to generate $q\bar{q}$ events at the center of mass energy of 91.2 GeV. Due to the low efficiency of neutral particle reconstruction in e^+e^- experiments, the study of the Bose-Einstein effect was performed only for charged particles and for subsamples of charged pions, pions that were produced directly from the hadronization of quarks and gluons, pions coming from resonance decays and separately of pions that appeared in decays of ρ mesons which were in turn produced in the hadronization of the string (direct ρ's). Different values of the JETSET input radius R (see (4)) are used for studies.

4.1 The reference sample

In order to prepare the reference sample for a proper correlation function (see Sect. 2.1), we have to find a sample of particles which are not subject to Bose-Einstein correlation, but do obey the same kinematics as a regular e^+e^- event.

There are several procedures to prepare a reference sample. For this study, as, for example, in [10,11], we combined particles from different events, assuming that the selection criteria of the two-jet events provide us with a set of kinematically similar events. This so-called "mixing" procedure can be described by following steps:

– After the thrust[2] axis calculation, each event is rotated to a new coordinate system, which has the z axis along the thrust axis.

[2] The quantity thrust T is defined by:

$$T = \max_{|\mathbf{n}|=1} \frac{\sum_i |\mathbf{n}\cdot\mathbf{p}_i|}{\sum_i |\mathbf{p}_i|} \ ,$$

and the thrust axis $\mathbf{v}_1$ is given by the $\mathbf{n}$ vector for which the maximum is obtained. The allowed range is $1/2 \leq T \leq 1$, with a 2-jet event corresponding to $T \approx 1$ and an isotropic event to $T \approx 1/2$

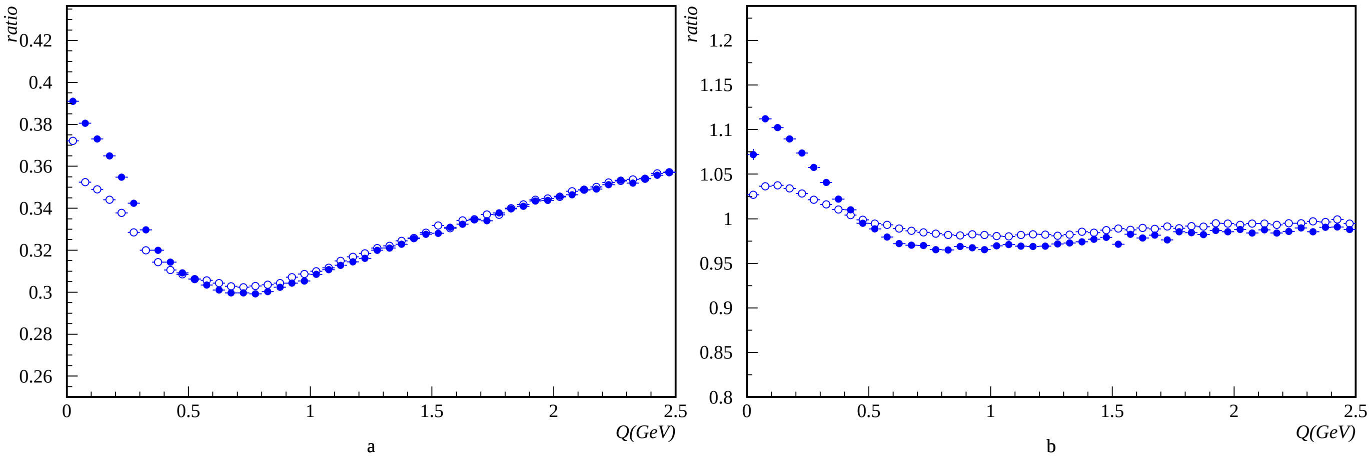

Fig. 2. **a** The ratios $N_{BEon}(Q)/N^{MIX}_{BEon}(Q)$ (closed circles) and $N_{BEoff}(Q)/N^{MIX}_{BEoff}(Q)$ (open circles), **b** The ratios $N_{BEon}(Q)/N_{BEoff}(Q)$ (closed circles) and $N^{MIX}_{BEon}(Q)/N^{MIX}_{BEoff}(Q)$ (open circles)

– Tracks from each rotated event are stored in a reference buffer. Events in the buffer are continuously updated to prevent any regularities in the particles spectra.
– The reference sample is built using randomly picked tracks from the reference buffer. First, a random event of the stored ones is selected, then a track from this event is also randomly picked out.

The mixing procedure does not conserve energy and momentum, and destroys not only the Bose-Einstein correlation but even some other kinds of correlations, making necessary some corrections.

4.2 The correction procedure

To construct properly the correlation function, eliminating side effects introduced by the mixing procedure, we accumulate four kinds of number of charge-like pairs N as a function of Q:

– Bose-Einstein correlation effect turned on, but the mixing procedure not performed: N_{BEon}, see Fig. 1a (closed circles).
– Bose-Einstein correlation effect turned on, and the mixing procedure performed: N^{MIX}_{BEon}, see Fig. 1a (open circles).
– Bose-Einstein effect turned off, and the mixing procedure performed: N^{MIX}_{BEoff}, see Fig. 1b (open circles).
– Bose-Einstein effect turned off, and the mixing procedure not performed: N_{BEoff}, see Fig. 1b (closed circles).

The corrected correlation function is given by:

$$C_2(Q) = \frac{N_{BEon}(Q)/N^{MIX}_{BEon}(Q)}{N_{BEoff}(Q)/N^{MIX}_{BEoff}(Q)} \ . \tag{7}$$

In this way we get $C_2(Q)$ almost constant for $Q > 1$ GeV, as we show below (see Fig. 3).

Figure 2a shows that the ratios $N_{BEon}(Q)/N^{MIX}_{BEon}(Q)$ and $N_{BEoff}(Q)/N^{MIX}_{BEoff}(Q)$ are very similar in shape.

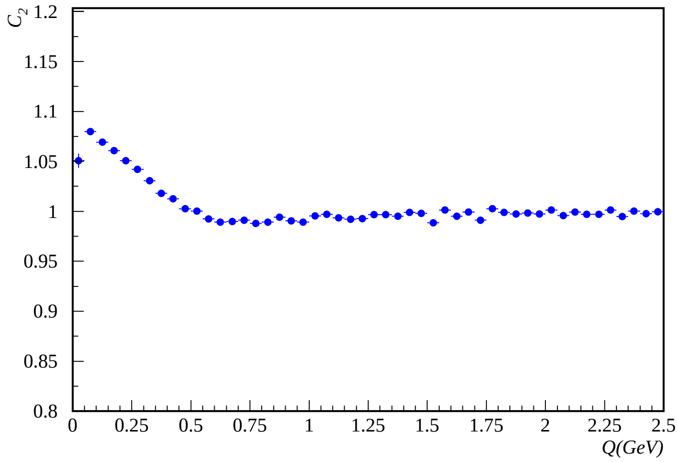

Fig. 3. The final corrected correlation function, as in (7)

Even in the $N_{BEoff}(Q)/N^{MIX}_{BEoff}(Q)$ case, a small enhancement, that has approximately the same width, appears due to the other kinds of correlation. In Fig. 2b, the ratio $N^{MIX}_{BEon}(Q)/N^{MIX}_{BEoff}(Q)$ shows that the mixing procedure does not eliminate correlations totally. This is why we have to calculate the double ratio (7) to obtain a clear Bose-Einstein effect over the $C_2(Q) = 1$ level, see Fig. 3.

4.3 Event and track selection

To be able to test the string model and to compare with the previous experimental data, we generate $q\bar{q}$ events only, requiring also the thrust value to be bigger than 0.95. In order to purify the data samples, to reduce the background and to save computing resources, the following cuts were introduced:

– The correlation function was constructed only for pairs of particles belonging to the same jet , each having momentum below 5 GeV/c to avoid the limits of phase space, where dynamical correlations are strong.

Table 1. Number of events analysed. Samples of pions coming from direct ρ were collected for five different values of input correlation function radii R

Sample composition	Number of events	Average multiplicity
Charged particles	2 000 000	14
Pions	2 000 000	12
Direct pions	5 000 000	2
Pions from all resonances	5 000 000	6
Pions from direct ρ	50 000 000	3

Table 2. Results of the one-dimensional fit of $C_2(Q)$

Sample composition	λ	R (fm)	χ^2/ndf
Charged particles	0.106±0.005	0.531±0.018	26/25
Charged pions	0.132±0.005	0.534±0.008	37/25
Direct pions	0.950±0.039	0.602±0.018	39/25
Pions from all resonances	0.656±0.010	0.651±0.008	111/25
Pions from direct ρ	1.090±0.010	0.665±0.005	160/25
Input values	1	0.5	

– To reduce correlations due to the local transverse momentum compensation, the pairs were rejected if their opening angle in transverse plane exceeded 120°. This cut was introduced for the compatibility with experimental data and reduces slightly the background at high Q values.

Since the number of pairs of particles that accomplished all our cuts is small (especially in the case of mixed samples for pions from resonances and direct pions, where the multiplicity is low) it was necessary to analyse large samples (see Table 1).

5 The dependence on the particle origin

The correlation function (7), obtained with the help of JETSET, was fitted in the interval of 0.05 GeV $< Q$ < 1.5 GeV, using formula (2) in order to find its parameters. These output fit parameters, λ and R, are given in Table 2 for different sample composition.

Table 2 shows that the simulated correlation functions are not always well described by a Gaussian. The output values of parameters are quite different not only from the input values, but also from case to case.

The λ parameter represents a measure of the fraction of particles sensitive to the effect. In our case it is easy to understand that for charged particles and for pions, where the particles have different origins, λ has low values. The value of this parameter in the case of pions coming from all resonances is also far from the input value, even if it is considerably higher compared to the one for charged particles and charged pions. This is due to the fact that there are some pions coming from long-lived resonances and thus are not subject to Bose-Einstein symmetrization.

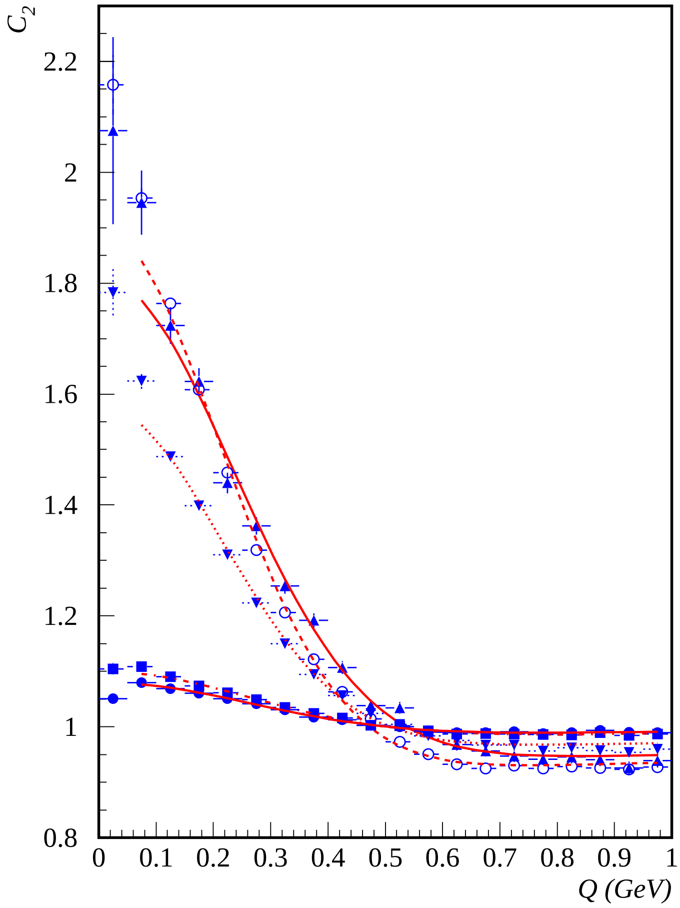

Fig. 4. The shape of the correlation function in different cases: all charged particles (closed circles), pions (squares), direct pions (up triangles), pions coming from all the resonances decay (down triangles), pions from direct ρ (open circles)

In JETSET a corresponding mechanism is implemented that allow to ignore them. In case of direct pions and pions coming from ρ decay, where the pion source has a high purity, the fitted values of λ are close to the input value $\lambda = 1$. For the radii, we obtained fitted values which are 20% to 40% higher than the input.

The correlation function $C_2(Q)$ at $Q > 0.5\,GeV$ eventually becomes smaller than one. The shape of the resulting correlation function is not particularly close to a Gaussian at very low Q, (see Fig. 4).

6 The dependence on input radius

Following the study of Fiałkowski and Wit [12], we studied how the correlation changes with the input JETSET radius. The shape of the functions changes from the input Gaussian due to the global shift of the momenta, the adding of all the pair shifts and the poor approximation of the phase space for input radius of $\sim 0.5 fm$.

For this study we used only samples of pions coming from the direct ρ decays, since this sample is relatively pure and the multiplicity is higher than for direct pions. It is easy to observe, Fig. 5, that when the input radius

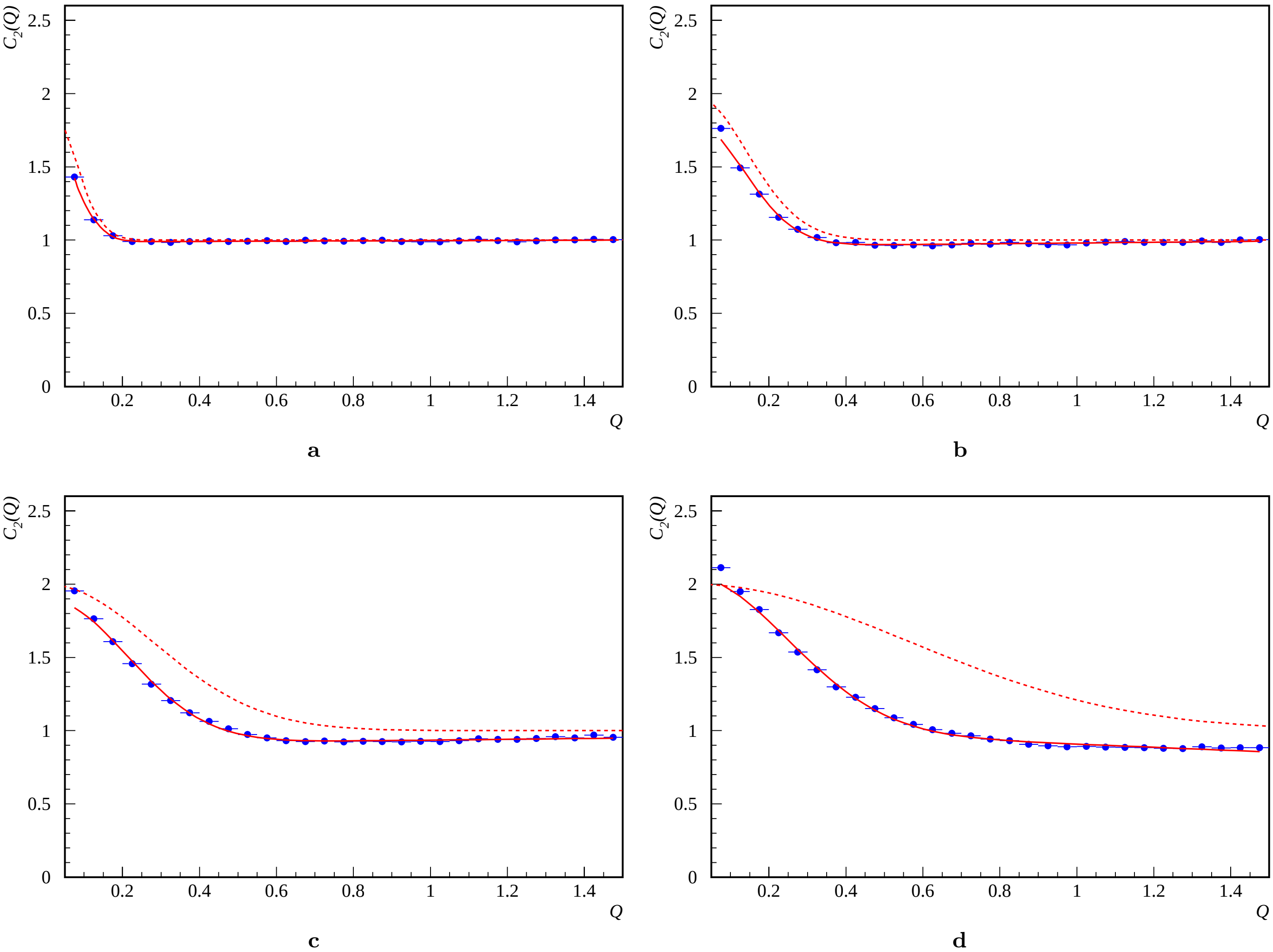

Fig. 5. The shapes of the output correlation function (full curve) and the input correlation function (dashed curve) for different input radii: **a** R_{in} =2 fm, **b** R_{in} =1 fm, **c** R_{in} =0.5 fm, **d** R_{in} =0.25 fm

decreases, the shape of the output function (full curve) is more and more different from the shape of the input function (dashed curve).

Figure 6 represents the radii R_o obtained through the fit by (2), as a function of the input radii R_{in} for JETSET. One can observe that for the values of input radii less than 0.5 fm, almost constant values of the output fitted radii were found, around 0.6 fm. For input R_{in} values bigger than 1 fm, the points are lying very close to the diagonal, showing a good agreement between input and output radii.

We observe the appearance of a new length scale of the resulting radius, $R_o \sim 0.6\,fm$, which is independent of the input radius R_{in}, for the region $0 < R_{in} \leq 0.6\,fm$. This new scale is connected to the peak value of the inclusive Q distribution. In the studied algorithm, Q is shifted to a lower value Q' to give an enhancement when we take the ratio of the Q' distribution to the Q distribution. This works only for a monotonically increasing function, such as the spherical phase space distribution. The true Q distribution, however, exhibits a peak around 0.5 GeV (which

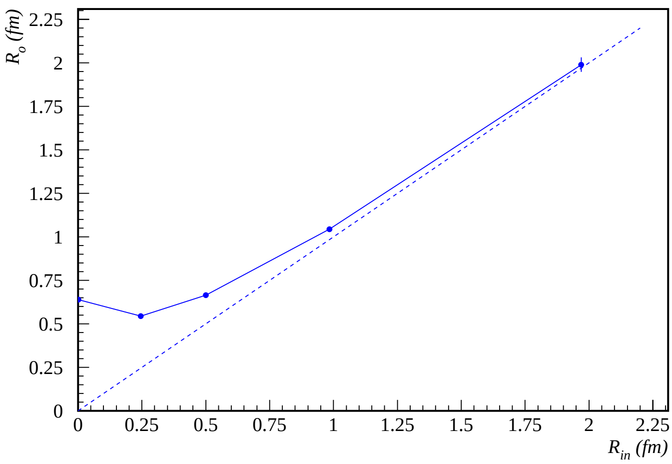

Fig. 6. Fitted values of output radii R_o for different input radii R_{in}

corresponds to $2.5\,fm$). For $R_{in} \leq 0.6\,fm$, the shift of Q to a lower Q' implies a depletion, not an enhancement in

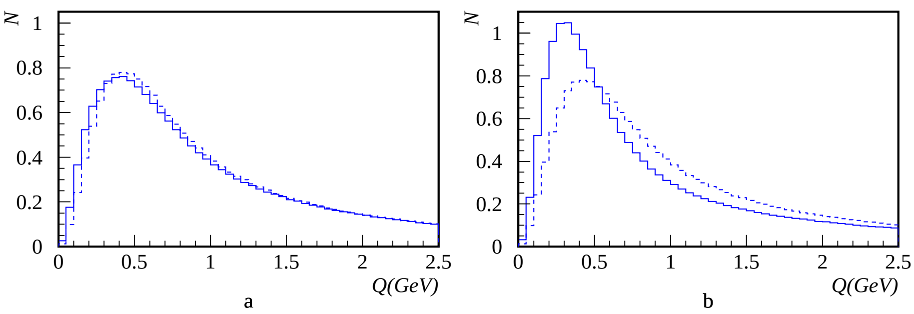

Fig. 7. The Q distribution with Bose-Einstein simulation (full line), and without it (dashed line) for **a** $R_{in} = 1 fm$, **b** $R_{in} = 0.25 fm$. All the distributions are normalised to the total number of pairs

the region to the right of the peak ($Q > 0.5\ GeV$), see Fig. 7, possibly giving a $C_2 < 1$ for this region, as seen in Fig. 5c and Fig. 5d. For the Q distribution, the peak position constitutes a limitation of the correlation width and gives rise to an artificial length scale.

To summarise, for the input radius values higher than $1\,fm$, the input correlation function seems to be well reproduced by the constructed output correlations. For lower input radii secondary effects, due to the way in which the Bose-Einstein correlation is introduced, are important. This is consistent with earlier results from other studies [12,13]. It is worth noting that experimental data seems to be reproduced by this secondary effects simulation using $R_{in} = 0.5\,fm$ [6,9–11].

7 Conclusions

An analysis of the Bose-Einstein correlation, implemented in the JETSET particle generator, has been performed. Our main result is that an artificial new length scale is introduced due to the way the generator is working. This scale is given by the peak position of the Q distribution. Appearance of this artificial scale causes the previously unexplained phenomenon; namely, the fact that JETSET, while reproducing well experimental data, is self-inconsistent and fails to reproduce its own built-in Bose-Einstein correlation function for input radii of around 0.5 fm. For radii larger than 1 fm we have not seen any strong artificial scale effects.

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